DYNAMIC INVENTORY CONTROL SYSTEM WITH LEAD-TIME UNCERTAINTY: ANALYSIS AND EMPIRICAL INVESTIGATION

M. Zied Babai and Aris A. Syntetos
Centre for Operational Research and Applied Statistics (CORAS), Salford Business School,
University of Salford, Maxwell Building, Greater Manchester M5 4WT, UK.
Email: m.z.babai@salford.ac.uk, a.syntetos@salford.ac.uk

ABSTRACT
A new forecast-based dynamic inventory control approach is discussed in this paper. In this approach, forecasts and forecast uncertainties are assumed to be exogenous data known in advance at each period over a fixed horizon. The control parameters are derived by using a sequential optimization procedure. The merits of this approach as compared to the classical one are presented. We focus on a single-stage and single-item inventory system with non-stationary demand and lead time uncertainty. A dynamic re-order point control policy is analyzed based on the new approach and its parameters are determined for a given target cycle service level. The performance of this policy is assessed by means of empirical experimentation on a large demand data set from the pharmaceutical industry. The empirical results demonstrate the benefits arising from using such a policy and allow insights to be gained on other pertinent managerial issues.

KEY WORDS
Stock Control, Forecasting, Lead Times, MRP, Simulation, Empirical Experiment

1. Introduction
The efficient management of inventories is an important concern of all companies that deal with physical stock at any level of a given supply chain. Inventory control is viewed as an important task since it is the lever enabling significant cost reductions and/or higher customer service levels. This task becomes more complex in the context of highly variable customer demand.

Many academic investigations have been performed since the 1930’s in order to develop or to analyze new inventory control policies that can be implemented in real inventory management systems. It should be noted that in the context of highly variable demand, the standard “static” control policies such as the static re-order point or the static order-up-to-level policy, often are not appropriate since they are based on the stationary demand assumption. It proves to be essential to develop dynamic inventory control policies considering non-stationary demand since that resembles better to the variable patterns encountered in practice.

It is also important to note that in practice, numerous companies are still using static policies to compute the ordering quantities and safety stocks for non-stationary independent demands in MRP systems. These quantities are often derived based on histories of demand and they are rarely reviewed. This is the case even when demand forecasts are available. Hence, dynamic inventory control systems aiming to tackle this issue have to be developed in order to help practitioners in their decision making process, especially when using MRP systems.

Some academic research work has been developed that deals with dynamic inventory control policies under non-stationary demand (with or without the use of forecasts). However, the methods developed in the relevant work are either not easy to implement by practitioners (i.e. computationally intensive and/or not intuitively appealing or they have been developed upon particularly restrictive assumptions thus making their implementation strongly dependant upon specific underlying demand patterns and forecasting methods.

In this paper a new approach to forecast-based inventory control is discussed. Our aim is to develop a simple dynamic inventory control policy that can be easily implemented in practice, under a non-stationary demand pattern that is estimated by any forecasting procedure. We assume that forecasts and probability distributions of forecast uncertainties are exogenous data known in advance at each period over a fixed horizon. Our approach avoids the explicit reliance upon any underlying statistical model based on which the forecasts may be derived. As such, it allows the consideration of forecasts derived based on ad-hoc methodologies or forecasts generated by specialised statistical forecasting software (in which case the exact updating algorithms are rarely known). Moreover, our approach enables experimentation with judgementally adjusted or qualitative forecasts that are generated upon relevant management and/or marketing intelligence. Such forecasting procedures are often used in practice. Nevertheless, their ‘arbitrary’
nature is often found to be very constraining in terms of their integration in any theoretically sound inventory control policy. That is to say, in all those cases the underlying mechanism based on which the forecasts are derived is not known. However, the forecasts themselves could still be utilised/integrated in a structured manner for the purpose of controlling inventories. Our approach enables real-world systems to do so. Please note that in practice, our approach is compatible not only with MRP systems for independent demands (demands for items in the level zero of the Bill of Materials)\(^1\), but also with decentralized forecast-based inventory control systems.

In previous work by [3], some preliminary work on the above discussed approach was developed. In this paper we extend the analysis by considering a lead time uncertainty and by assessing the performance of a stock control policy (namely the re-order point one) by means of an empirical experimentation on a large demand data set from the pharmaceutical industry. Two models of forecast uncertainty are considered, namely: an absolute model and a relative one. Experimentation with the latter model constitutes a contribution on its own since not much work has been done in this area.

The remainder of this paper is organized as follows: in Section 2, we propose a classification of inventory control policies based on the demand information available for decision making. Dynamic approaches to inventory control are subsequently reviewed for the purpose of contrasting them with our approach. In Section 3, the assumptions upon which our work is built are discussed in detail followed by the development of the dynamic inventory control policy. The experimental structure of our empirical investigation, along with a brief description of the demand data set available for simulation purposes, is presented in Section 4. The empirical results are analyzed in Section 5 and, finally, the conclusions of this research along with some natural extensions for further work are given in Section 6.

2. Inventory Control Policies

In this section, we first provide a classification of various inventory control policies for the purpose of communicating in a comprehensive manner where our work fits. The dynamic policies are then further considered in order to provide the background for the development of our approach.

---

1 In such approaches, estimation of required quantities is needed over a long horizon to allow the backwards explosion of the Bill of Materials (BoM).

2.1 Classification of Inventory Control Policies

Inventory control policies can be classified into two categories according to the type of the demand information available. The first category, that we refer to as ‘inventory consumption based policies’, may be applied when demand is stationary and when there is no advance demand information. The parameters of the corresponding inventory control policies are static and computed by considering a known and fixed probability demand distribution derived from the available demand history. Once the control parameters are determined, based on the demand distribution, decisions are then being made in real time based on the inventory consumption.

This class of policies consists of the standard inventory control policies (e.g. the re-order point policy or the order-up-to-level one) operating with static control parameters ([19], [25]) and the Kanban based policies, e.g. the standard Kanban policy, the extended Kanban and the generalized Kanban policy ([5], [15], [16]). In the latter case, the number of Kanbans is computed by considering a probability demand distribution derived from the demand history. The second category, referred to as ‘future requirements based policies’, may be applied when demand is non-stationary and advance demand information is available. The advance demand information can be in the form of firm orders and/or demand forecasts. In this class of policies, we include the MRP type control policies ([2], [6], [11], [17], [23]) and the forecast-based dynamic control policies ([7], [8], [10], [20]). The literature related to the forecast-based dynamic control policies is reviewed in more detail in the following subsection. Our classification of inventory control policies is summarized in Figure 1.

Other classifications of inventory control policies have been proposed in the academic literature in order to conceptually distinguish between them and enable a better understanding of their comparative advantages and disadvantages. The most common ones are the ‘Pull vs. Push’ and ‘Make-To-Stock (MTS) vs. Make-To-Order (MTO)’ classifications. For more details, the reader is referred to [4], [11] and [14], [16] discussed in detail the potential inconsistency resulting from such classifications and the controversy associated with them that often renders their contribution ambiguous. Such classifications rely heavily on the definition employed for characterising the relevant practices and they are very much dependent on the context of application. For example, MRP may be only classified as either ‘Push’ or ‘Pull’ (MTO or MTS) depending on the relevant definitions that do not allow a more holistic view of the relevant practices. We show below that our classification approach avoids such drawbacks and in fact encompasses the above discussed classifications.
Indeed, if there is advance demand information in the form of firm orders, allowing activities to be initiated before demand occurs, there is less need to 'blindly' set inventories ahead of time. This is typically the case in Make-To-Order policies, such as the MRP type control policies when they incorporate safety times. If there is no advance demand information or demand forecasts, the control policies need to set inventories in order to cope with the uncertainty. This is typically the case with Make-To-Stock policies. Please note that the MRP policy is a Make-To-Order policy that copes with uncertainty, usually by inflating lead times rather than by introducing an inventory target level in the form of safety stock. However, if safety stocks are set when forecasts are available, it can also be considered as a Make-To-Stock policy. A discussion on issues related to safety stock versus safety time in MRP systems is given by [6].

Furthermore, it can also be claimed that all the control policies discussed above may be viewed as pull control systems. Indeed, in the MRP type control policies, the system is pulled by the firm orders. In the forecast-based policies the system is pulled by forecasts, and in both standard inventory policies (re-order point and order-up-to-level) and the Kanban based policies, the system is pulled by the real inventory consumption.

Our contribution at this stage is purely conceptual and it is not further elaborated in our paper. The classification scheme enables only, as discussed in the introduction of this section, a better understanding of where our proposed stock control approach fits on the body of literature. In the following subsection, we focus on forecast-based dynamic inventory control policies.

### 2.2 Dynamic Inventory Control Policies

There are two main streams of literature dealing with dynamic inventory control policies for non-stationary demand. The first one is based on the earlier work by [9], [13] and [18]. This literature investigates optimal inventory control policies with an objective to minimize total inventory costs including backlog penalties. It considers non-stationary stochastic demands over periods of an infinite horizon. It shows that optimal control is obtained in the form of dynamic \((s, S)\) policies, where \(s\) is the re-order point and \(S\) is the order-up-to-level. Nevertheless, the relevant parameters cannot be determined easily. That is to say, the proposed solutions are difficult to understand from a practitioner perspective and difficult to implement (computationally intensive/demanding). Moreover, the optimal policies have been developed based on the minimization of the total inventory cost function under restrictive assumptions. Often systems operate under a service level constraint and should this be the case a different approach is needed.
More recently, a body of literature studying forecast-based dynamic inventory control policies has also been developed. Interested readers are referred to [7], [8] and [10]. The related work considers auto-correlated non-stationary demand (often for fast moving items), where forecasts are obtained by using statistical forecasting methods. It subsequently develops dynamic order-up-to-level policies where optimal parameters are computed under service level constraints. Nevertheless, there is an implicit exclusion of forecasts derived based on ad-hoc methodologies and/or in a qualitative manner (judgemental forecasts). In addition, statistical forecasts are often integrated with marketing intelligence and should this be the case, final forecasts (i.e. statistical forecast + judgemental adjustment) resemble very little to the original statistically derived ones [22]. Finally, all the above discussed work has also been generated upon restrictive assumption and does not take into account important practical concerns, such as the lead-time uncertainty.

In this paper, we propose a simple and realistic approach that addresses the practical needs of inventory managers in the context of non-stationary demand by considering lead time uncertainty. The advance demand information is given by forecasts and the objective is to satisfy a target service level. The new forecast-based inventory control approach addresses the drawbacks of the classical approaches cited above. In fact, we assume that forecasts and the probability distributions of the forecast uncertainties are exogenous data known in advance over a fixed horizon. This approach works for any forecasting procedure. In order to derive the parameters of a given control policy, say the re-order point one, we use a sequential optimization procedure under a service level constraint, which means that the value of the ordering quantity is computed ignoring the impact upon the re-order point. In order to derive the re-order point, a target cycle service level, CSL, is set which represents the probability of no stock-out during a cycle (a cycle is the time period between two successive orders).

This approach has three merits as compared to the other approaches cited above: i) It is generic since the resulting control policy does not depend upon any forecasting procedure/parameters. Consequently, qualitatively adjusted forecasts (based on management/marketing intelligence) may also be considered. Please note that in those cases there is not an explicit statistical model based on which the forecasts are derived; ii) It is less computationally intensive as compared with other dynamic approaches proposed in the literature - such approaches deliver optimal results under very specific assumptions; iii) It is compatible with MRP systems and it can be used to compute the ordering and safety parameters in relevant applications. It also allows for the integration and experimentation with various alternatives in analyzing real systems, such as the lead time uncertainty.

Please note that if forecasts are obtained by using only statistical forecasting methods and the probability distributions of forecast uncertainties are obtained by considering the forecast errors given by the underlying forecasting model, the classical approach of forecast-based inventory control collapses to our approach.

3 In the classical approach, the forecast error is often represented by the Mean Square Error or the Mean Absolute Deviation.

3. Assumptions and Model Development

3.1 System Description and Assumptions

We analyze a single-stage and single-item inventory control system under non-stationary demand, information of which is provided through forecasting. The system is not capacitated and the inventory replenishment requires a lead-time, as shown in Figure 2.

![Figure 2: The Inventory System Model](image)

We assume that the replenishment lead time $L$ is a random variable. Since, we analyze a discrete time control system, we assume that the lead time $L$ is given by a discrete probability distribution (i.e. $L$ takes on the values $L_i$, such as $\sum_i P(L = L_i) = 1$) with a mean and standard deviation denoted by $\mu_L$ and $\sigma_L$ respectively.

---

2 Some literature analyzing forecast-based dynamic inventory control policies for intermittent demand items includes [20] and [21].
Two key elements must be available as exogenous data just before the beginning of the forecast horizon in order to estimate future requirements and to set the control parameters, as shown in Figure 3: i) the point forecasts over all periods in the forecast horizon; ii) a Probability Distribution Function (PDF) for the forecast uncertainty (the often so-called Error Distribution).

Please note that for the purpose of this research, the uncertainty associated with the provided forecasts is referred to as the ‘forecast uncertainty’, whereas other authors often refer to it as the ‘forecast error’. We adopt this terminology since we assume that the relevant information is an exogenous indicator of the uncertainty involved in the provided demand forecasts and it is not only the error resulting from a statistical forecasting method.

Generally, the random forecast uncertainty can be absolute (or additive), relative (or multiplicative) or mixed. The forecast uncertainty is absolute if it is independent of the forecasts. It is relative, if it is proportional to the forecasts and it is mixed if it consists of both an absolute and a relative component. In this paper, we assume that the forecast uncertainty may be either absolute or relative. Hence, for a period $k$, if $F_k$ denotes the forecast produced for the demand in that period and $FU$ denotes the random forecast uncertainty associated with that estimate, a representation of the actual demand, denoted by $D_k$, is given by:

$$D_k = \begin{cases} F_k + FU & \text{If the forecast uncertainty is absolute} \\ (1 + FU)F_k & \text{If the forecast uncertainty is relative} \end{cases}$$

Please note that if the forecast uncertainty is relative, the PDF is given in percentages (i.e. the mean and the standard deviation of the forecast uncertainty as well as the realizations are expressed in percentages).

To the best of our knowledge, most of the literature that deals with forecast-based inventory control policies is developed by considering absolute forecast uncertainties. However, it is important to mention that in many cases encountered in practice, the forecast uncertainty for some SKUs may be better represented by a relative or a mixed model.

In inventory control, the safety stock covers the cumulative forecast uncertainty over the protection interval which, in periodic review systems, is equal to the lead-time plus one review period. Therefore, the PDF of the cumulative forecast uncertainty over the protection interval is needed. The literature suggests that the common way to do this is by aggregating the PDFs of the forecast uncertainties associated with each period over the protection interval [1]. However this approach - although common in practice - is not intuitively appealing as it does not allow the consideration of autocorrelations of forecast uncertainties over the protection interval; this could only be achieved via a true Cumulative PDF, and this is what we suggest that it should be adopted. Hence, we want to make the point that a PDF that could handle an interval $R$ composed of multiple periods as one ‘large’ period, should lead to lower inventories and thus is regarded as a more cost-effective solution.

Let us assume now that forecasts and the PDF of the cumulative forecast uncertainty over any interval $R$ (interval composed of $R$ periods) are known at each period of the horizon. We also assume that the cumulative forecast uncertainty over any interval $R$ is Normally distributed. The Normal assumption is reasonable for fast moving items (the ‘Class A’ items in a typical Pareto-Based Classification of SKUs) and for slower moving items with large replenishment lead times (because of the Central Limit Theorem). For the remainder of our paper, we denote by:

$F_{k,j}$: the forecast given at the end of period $k$ for period $j$ ($j \geq k + 1$)

$CFD_R$: the random cumulative forecast uncertainty over an interval $R$

$\Phi_{CFD_R}()$: the probability distribution function of $CFD_R$

$\mu_{CFD_R}$: the mean of the cumulative forecast uncertainty $CFD_R$

$\sigma_{CFD_R}$: the standard deviation of the cumulative forecast uncertainty $CFD_R$
3.2 The \((r_k,Q)\) Policy with Lead-Time Uncertainty

In this subsection, we consider the system described above for the dynamic periodic re-order point \((r_k,Q)\) control policy. Under this policy, the system is controlled at the end of every review period. At the end of each period \(k-1\), if the inventory position falls below the re-order point \(r_k\), a quantity \(Q\) is ordered. The quantity ordered is received after \(L\) periods. The inventory level evolution in the \((r_k,Q)\) policy is graphically presented in Figure 4.

By using a sequential optimization procedure, the optimal ordered quantity is computed independently of the lead time uncertainty. The ordering quantity \(Q\) can be computed based on the constraints and characteristics of the inventory system. It can also be optimized once and for all (at the beginning of the horizon) by balancing the inventory holding costs and the ordering costs by using for example the Wilson’s formula as follows:

\[
Q = \sqrt{2A \sum_{j=1}^{L} F_{0,j}}\frac{1}{hH}
\]

The quantity \(Q\) can also be dynamically optimized based on more complex methods such as the Wagner-Whitin method [24].

The re-order point \(r_k\) can be computed numerically, at the end of period \(k-1\), by resolving the equation:

\[
\sum_{i} P(L = L_i) \Phi_{CFD_{k-1,L_j+1}}(r_k) = CSL
\]

where \(CFD_{k-1,L_j+1}\) denotes the sum of the cumulative forecasts plus cumulative forecast uncertainty over \(L_i+1\) periods, estimated at the end of period \(k-1\) (i.e. \(CFD_{k-1,L_j+1} = CFU_{L_j} + \sum_{j=1}^{L_i} F_{k-1,j+1} \cdot \Phi_{CFD_{k-1,L_j+1}}(...)\)) its cumulative probability distribution.

The proof that shows the expression of \(r_k\) may be provided by the first author upon request.

4. Empirical Investigation

In this section, we present a brief description of the demand data set available for the purposes of this research, along with the experimental structure of our empirical investigation.

4.1 Empirical Data

The empirical investigation is based on data relating to monthly demand forecasts for the UK branch of a major international pharmaceutical company. The company relies upon a commercially available software to produce system forecasts per SKU for each time period (i.e. month). Final forecasts are produced at a later stage through the superimposition of qualitative judgements based on marketing intelligence given by the company forecasters.

The database consists of the individual demand histories of 829 SKUs. Distinct demand patterns are included in the sample, i.e. lumpy, intermittent and smooth demand patterns. Demand is recorded monthly and the available history covers 36 consecutive periods from January 2003 to December 2005 (inclusive three years).

System forecasts are available for all time periods and over an horizon of 36 periods (36 steps ahead forecast). In addition, the judgemental adjustment is also available, where applicable (with a sign, i.e. plus or minus). The statistical forecast provided by the software plus the judgemental adjustment gives the final forecast, i.e. the one used for decision making purposes.
Not all series were considered for experimentation purposes. We exclude from our empirical investigation series with missing demand data or invalid recording of data; intermittent demand series (i.e. series with some streaks of zeroes); series that consist of a streak of zeroes followed by a streak of non-zero demands or the other way around (new SKUs or re-coded ones respectively). It is important to note that there have been series containing some negative values (returns). We have opted for retaining the relevant information by replacing the negative values with zero (see also [22]).

The screening process resulted in 135 files being considered for our simulation purposes. All of them are fast moving products. More details on the demand characteristics across all SKUs are given in Table 1.

### Table 1: Demand Data Descriptive Statistics (Across All SKUs)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>15.03</td>
<td>13.59</td>
<td>0.14</td>
</tr>
<tr>
<td>25%ile</td>
<td>219.36</td>
<td>107.89</td>
<td>0.25</td>
</tr>
<tr>
<td>Median</td>
<td>605.56</td>
<td>252.68</td>
<td>0.49</td>
</tr>
<tr>
<td>75%ile</td>
<td>1503.33</td>
<td>481.95</td>
<td>0.74</td>
</tr>
<tr>
<td>Max.</td>
<td>4366.17</td>
<td>3099.14</td>
<td>1.97</td>
</tr>
</tbody>
</table>

### 4.2 Simulation Details

We use the demand data set described above to empirically assess the performance of the dynamic \((r_k, Q)\) policy against that of the commonly employed static reorder point policy \((r, Q)\). In the former case the computed re-order points vary over the forecast horizon, depending on the relevant forecasts (which are available for the purposes of our research), whereas in the latter case one single re-order point is set to control inventories. That is to say, in the \((r, Q)\) policy, we assume that there is no advance demand information used to control inventories and the parameters of the policy are set once and for all based on some historical demand information. We split the demand history available (i.e. 36 periods) into two parts. The first part, referred to as ‘within-sample’, is composed of \(N_1\) periods, and is used as the history of demand required in order to determine the probability distributions of forecast uncertainties for the \((r_k, Q)\) policy and the determination of both \(r\) and \(Q\) in the \((r, Q)\) policy. The second part, referred to as the ‘out-of-sample’, is composed of \(N_2 = 36-N_1\) periods and it is used in order to evaluate performance for both policies. For example, if \(N_1 = 21\), we use a within-sample of 21 periods from January 2003 to September 2004 to determine the probability distributions of forecast uncertainties. In our simulations in order to evaluate their impact on the performance of the \((r_k, Q)\) policy.

Regarding the width of within-sample block, there is obviously a trade-off between having sufficient information in order to compute the parameters of the PDF of the cumulative forecast uncertainty and having enough time periods in order to evaluate the out-of-sample performance. In other words, ‘few’ periods for initialisation would lead to a rather simplistic representation of the forecast uncertainty. Correspondingly, ‘few’ periods in the out-of-sample evaluation, would not allow us to gain sufficient insight on the comparison result. Therefore, three widths of the within-sample are considered, \(N_1 = 15, 18, 21\) which corresponds to \(N_2 = 21, 18, 15\) periods (respectively). Nevertheless, and due to space restrictions in this paper results are provided for the scenario \(N_1 = 18\) and \(N_2 = 18\) periods which represents the best trade off between the width of the within-sample and out-of-sample blocks of time.

Regarding the lead-time length, it is important to note that recent developments in transportation imply that lead times have generally been dramatically reduced. In the context of the pharmaceutical industry in particular, and given the fast moving nature of that industry, lead-times are ‘fairly’ short. In the case of the particular company that provided the empirical data, lead-times generally do not exceed twelve weeks (i.e. 3 months). Short lead times are also dictated for the purposes of our simulated experiments by the demand history available. That is to say, the cumulative evaluation of forecasts reduces the number of periods that may be considered for out-of-
sample evaluations. For example, if the out-of-sample is equal to 15 periods and the lead-time is equal to 3 periods, the evaluation may be performed only over 11 periods (i.e. 15- (L+1) periods).

Note also that in this paper we treat time as a discrete variable (months) and we want to introduce variability on the lead-time length. The stock control literature overall indicates that Normally distributed lead-times is a realistic assumption to make. In our case, the discretised Normal distribution could have been used in order to reflect the lead-time variability. Nevertheless, the short lead-times considered in our experiment would render such “mechanism” inappropriate (if lead-time is equal to 1, not many values of lead-times are possible, say for example 0, 2 and 3). Therefore, some probabilities are directly assigned to potential values of lead-times. Three discrete probability distributions are considered and the relevant details are summarised in Table 2.

In order to compute the \((r,Q)\) parameters: if \(\mu_D, \sigma_D\) and \(\mu_L, \sigma_L\) denote the means and the standard deviations of the demand and the lead-time respectively, then approximations of the parameters of the discrete time \((r,Q)\) policy, derived based on the sequential optimization procedure, are as follows:

\[
Q = \sqrt{\frac{2\mu_D}{h}} \quad \text{and} \quad r = \mu_D(\mu_L + 1) + \Phi^{-1}(\text{CSL})\sqrt{(\mu_L + 1)\sigma_D^2 + \sigma_L^2\mu_D^2}
\]

For the initialisation purposes, we assume that the initial stock in both the \((r,Q)\) policy and the \((r_S,Q)\) policy (stock at the end of the within-sample period) is given by:

\[
\text{Initial stock} = r = \mu_L(\mu_L + 1) + \Phi^{-1}(\text{CSL})\sqrt{(\mu_L + 1)\sigma_L^2 + \sigma_L^2\mu_D^2}
\]

The unit holding cost used in our experiments is \(h = 0.1 \text{ £/unit/period.}\) The ordering cost is \(A = 200 \text{ £/order.}\) Since the results in each scenario are dependant on the random realisation of lead times many replications need to be performed in order to ensure the 'stability' of the results given by simulation. The results reported in the following section have been obtained after 5 replications of each scenario.

### Table 2: Lead-Time Distributions

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
<th>Value</th>
<th>Probability</th>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.25</td>
<td>1</td>
<td>0.25</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>3</td>
<td>0.25</td>
<td>4</td>
<td>0.25</td>
</tr>
</tbody>
</table>

5. Empirical Results

The empirical results for all scenarios are summarised in Table 3 and Table 4.

The results indicate that, overall, as the lead time increases the achieved CSL decreases and the inventory cost increases. This is true for both policies considered in our experiment and this result is in accordance with theoretical expectations. That is to say, higher lead times (coupled with their stochastic nature, as simulated for the purposes of our experiment) imply higher uncertainty that is reflected on increased backlog and correspondingly increased inventory costs. Moreover, the results indicate that as the target CSL increases, the achieved CSL obviously increases as well (at the expense of a higher inventory cost) but it becomes more difficult to meet the actual target. For CSL = 0.8, 0.85 both policies offer CSL over and above the target ones. When the CSL = 0.90, the static policy continues to exceed the target; the same is true for the \((r_S,Q)\) policy when its performance is evaluated on short lead times. For longer lead times, the latter policy slightly under-achieves the target. Finally, for CSL = 0.95 both policies are found to be ‘unable’ to meet the target.

Overall, the average cost of \((r_S,Q)\) is lower than that of the static policy whereas the CSL is slightly higher in the latter case.
Table 3: Empirical Results for \( N_1 = 18 / N_2 = 18 \) (Absolute Forecast Uncertainty)

<table>
<thead>
<tr>
<th>( \mu_L = 3 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>42952.76</td>
<td>38295.96</td>
<td>45655.97</td>
<td>48706.86</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.892</td>
<td>0.858</td>
<td>0.891</td>
<td>0.879</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \mu_L = 2 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>40112.25</td>
<td>37222.26</td>
<td>42303.61</td>
<td>45313.48</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.892</td>
<td>0.877</td>
<td>0.904</td>
<td>0.892</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \mu_L = 1 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>37888.16</td>
<td>36001.33</td>
<td>40097.95</td>
<td>42866.12</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.899</td>
<td>0.890</td>
<td>0.911</td>
<td>0.906</td>
</tr>
</tbody>
</table>

Table 4: Empirical Results for \( N_1 = 18 / N_2 = 18 \) (Relative Forecast Uncertainty)

<table>
<thead>
<tr>
<th>( \mu_L = 3 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>42952.76</td>
<td>38295.96</td>
<td>45655.97</td>
<td>48706.86</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.892</td>
<td>0.858</td>
<td>0.891</td>
<td>0.879</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \mu_L = 2 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>40112.25</td>
<td>37222.26</td>
<td>42303.61</td>
<td>45313.48</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.892</td>
<td>0.877</td>
<td>0.904</td>
<td>0.892</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \mu_L = 1 )</th>
<th>Cost</th>
<th>CSL = 0.8</th>
<th>CSL = 0.85</th>
<th>CSL = 0.9</th>
<th>CSL = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(r,Q)</td>
<td>37888.16</td>
<td>36001.33</td>
<td>40097.95</td>
<td>42866.12</td>
</tr>
<tr>
<td></td>
<td>SL</td>
<td>0.899</td>
<td>0.884</td>
<td>0.911</td>
<td>0.905</td>
</tr>
</tbody>
</table>

Individualised attention was given to each of the SKUs considered in our experiment; the dynamic policy is consistently 'good' for each individual scenario whereas the performance of the static policy is highly variable. Consider for example a demand pattern characterised by a negative trend. In that case, the static re-order point set for that policy at the end of the \( N_1 \) period will certainly always exceed the true requirements resulting in CSL that exceed the target ones. Correspondingly of course, should a positive trend be present in the data the opposite will be the case. Since results are reported across all SKUs considerable differences cancel out, favouring the performance of the static policy. On the other hand, and as discussed above, the performance of the dynamic policy is consistently 'good' for each single SKU.

At this point we also need to remark that the performance of the dynamic policy is very much dependent upon the quality of the forecasts produced. High forecast uncertainty during the within-sample block of time and/or poor forecasts for the out-of-sample period render the performance of the \( (r,Q) \) policy (marginally) better than that of the \( (r_k,Q) \) policy in terms of achieved service level (whereas the cost related results still favour the dynamic policy). However, it is natural to assume that should better forecasts had been provided (resulting to a smaller forecast uncertainty), the \( (r_k,Q) \) policy would have out-weighted the \( (r,Q) \) one in terms of both costs and service level. This is what has been shown by [3] based on theoretically generated data for one SKU.

Overall, when all scenarios are considered (and when average results are computed across all parameter combinations), there is little to choose between the two policies in terms of the CSL achieves. Nevertheless, the dynamic policy results in inventory cost reductions that can be as high as 15%; that constitutes obviously a considerable difference for any real world system. Our analysis suggests quite conclusively that the adoption of the dynamic policy proposed in this paper should offer tangible benefits to manufacturers dealing with the problem of setting re-order points over a long horizon. This is particularly true in the context of any MRP application (under uncertain demand) in which case inventory control parameters need to be determined over a long term horizon to allow the backwards explosion of the Bill of Materials.

6. Conclusion

Computerised approaches to inventory management are constructed upon the assumption of various types of demand information available. Such information may be, most commonly, presented in terms of advance orders and/or forecasts of demand over a certain horizon. In this
paper, we have presented an overview of inventory control policies followed by the development of a classification scheme for those policies based on the type of available demand information. We have argued for the simplicity of this scheme that also serves the purpose of communicating in a comprehensive manner where our analytical work fits in the greater body of literature.

In this paper a new approach to dynamic forecast-based inventory control has been proposed. The approach has three merits: i) it is generic, since the determination of the control parameters does not depend upon any underlying demand structure and thus a corresponding 'optimal' forecast procedure; ii) it is not computationally intensive as compared with other dynamic approaches proposed in the literature, and iii) it is compatible with MRP systems operating at the independent demand level.

Following this approach, a periodic dynamic re-order point control policy has been developed, referred to as the \( r^*,Q \) policy. The parameters of the policy are provided under both demand and lead time uncertainty (variability) and for a cycle service level constraint. We showed that the re-order point \( r^* \) can be computed numerically by using, for example, a fixed point algorithm. Results are provided for an absolute forecast uncertainty model.

Subsequently, we have conducted an empirical investigation in order to evaluate the performance of the dynamic \( r^*,Q \) policy (relying upon advance demand information in terms of forecasts) in comparison with the static \( r,Q \) policy that is often employed in practical applications. Results have been generated for 135 SKUs from the Pharmaceutical industry, under a wide range of experimental conditions. The results indicate a rather similar performance of the two policies in terms of service level achieved. However, the significant inventory cost reductions obtained from the dynamic policy render its application preferable in any corresponding real world application.

An interesting line of further research would be to consider more elaborated models for representing the forecast uncertainty, such as the mixed model, as well as other possibilities of more regularly updating the demand information available. It would also be interesting to extend the analysis provided in this paper for other inventory control policies such as the order-up-to-level one. That in fact constitutes the next step of our research.

Acknowledgements

This research was supported by the Engineering and Physical Sciences Research Council (EPSRC, UK) grant EP/D062942/1. More information on this project may be found at:

http://www.mams.salford.ac.uk/CORAS/Projects/Forecasting/

References


