ABSTRACT
This paper describes a reverse supply chain from the point of view of a company providing after-sales service for the electronic products. The objective is to decide the facility locations and product flow to support two types of networks: a repair network for faulty products and refurbish network for commercial returns.

We propose a mixed integer linear programming (MILP) model to maximize total profit subject to conservation and flow constraints. The binary variables represent the use of facility for the networks while the continuous variables represent the amount of flow. The optimal decisions depend on several factors e.g. rate of product returned, percent of faulty products, demand of refurbished products, warranty, distance between facilities, and refurbishment fractions. The model will be tested on real-life data for validation.

KEYWORDS
Sustainable Supply Chain, Reverse Logistics, Network Design

1. Introduction
A considerable amount of electronic products are returned back to sellers after sales. They can be separated into two categories: (1) faulty returns where products are returned for repairing due to their malfunctions and (2) commercial returns where products are returned within their return period of 30, 60 or 90 days. They are returned not only by their imperfection but also compassionate reasons such as dissatisfaction and remorseful Guide et al. [4].

Refurbishment and repair are two of five recovery processes defined by Thierry [8]. In electronic products, the term “repair” generally means restoring faulty products to their working order. Repairing incurs at either original equipment manufacturers (OEMs) or repair vendors (RVs). On the other hand, “refurbishment” means bringing products to their original manufacturing specifications White and Naghibi [11].

The amount of commercial returns is around 12% of total new product sales [9]. Returned products in this category preserve value nearly as high as brand new one due to lightly uses in short period of time. Refurbishing and reselling these returns will generate very high profit due to the high margin between processing cost and price of refurbished products. Leading electronic product companies such as Dell, Apple, Nikon successfully implement refurbishment process and resale their returned products. In theory, Vorasayan and Ryan [10] show scenarios that refurbishment can be profitable and it can be a better option than selling dismantled modules as they are. Setting up reverse channel in an exiting forward channel is economically viable due to low set up cost and very reasonable because products acquisition and returns can be done in either retailers or service centers along with refurbishment can be carried out at the exiting facilities, in our case, e.g. service centers. The decision of using third party, exiting retailers or self collect for collecting used products has been previously studied by Savaskan [7]. Although the refurbished products might perform the same or nearly as well as new products, they are classified and priced lower than new products. The lower perceived quality of refurbished products comes from the view that they have been used and manufactured more than once. In this paper, we assume demand for refurbished products comes solely from the secondary market.
simple mathematical model was employed to determine the final network based on quantitative parameters. Other previous studies in reverse logistics are summarized in [1, 3].

Our study departs from previous works by concentrating on the networks for both repair and refurbishment processes. The optimal flows and configurations are decided based on several parameters such as the amount of supply, transportation cost, fraction of faulty modules under warranty, and faulty module which are sent for repair.

2. Problem description

Consider a reverse logistics network which deals with both commercial returns and faulty products as depicted in Figure 1. Customers interface with the network through retailers and service centers. They bring the commercial returns to the retailers to get partial or full refund. The retailers then forward the commercial returns to service centers where minor refurbishment is performed and the products are resold as refurbished products usually at a reduced price.

On the other side, faulty products are brought to the service centers for repair. Service centers identify the faulty modules and replace them onsite with new or repaired modules. To minimize costs, higher priority is given to the repaired modules over new modules during replacement. The faulty modules which are deemed repairable are then forwarded to distribution centers for consolidation. The distribution centers then forward the modules to OEMs or RVs. The OEMs and RVs repair the modules and send the fixed modules back to the distribution centers which are then forwarded to service centers for future use. The service centers may also buy new modules from OEMs to satisfy repair needs, as only a fraction of faulty modules may be repairable.

Locations and flows of facilities depend on the demand of refurbished products/repaired modules and supply of both kinds of returned products in regions. The choices of facilities for managing returned products are existing facilities in forward supply chain or new facilities dedicated solely for returned products. Using an existing facility reduces the setup costs while setting up a new facility can save transportation cost. In this paper, we propose a MILP model to determine a network configuration and flow between the facilities volumes that will generate the maximum profit from servicing these returns.

3. Mathematical model

We model the network as a mixed integer linear programming problem to determine optimal facilities locations and flows. The objective of the model is to maximize the net revenue while maintaining the flow balance, capacity, and logical constraints. The mathematical model is presented next.

Set Indices
- \( Q \) Regions
- \( T \) Time periods
- \( M_u \) Faulty and new modules in all periods
- \( N \) Nodes in the network
- \( N_c \) Feasible locations for retailers, \( N_c \subset N \)
- \( N_c^q \) Feasible locations of retailers in region \( q \), \( N_c^q \subset N_c \), \( q \in Q \)
- \( N_d \) Feasible locations for distribution centres, \( N_d \subset N \)
- \( N_d^c \) Potential locations for new distribution centres, \( N_d^c \subset N_d \)
- \( N_o \) Original equipment manufacturers, \( N_o \subset N \)
- \( N_o^m \) OEM for module \( m \), \( m \subset M_u \) and \( N_o^m \subset N_o \)
- \( N_r \) Feasible locations for repair centres, \( N_r \subset N \)
\( N^m_t \) Repair centres for module \( m \) in period \( t \), \( m \in M_u \) and \( N^m_t \subset N_t \cup (N_o - N^m_o) \)

\( N_s \) Feasible locations for service centres, \( N_s \subset N \)

\( N'_s \) Potential locations for new service centres, \( N'_s \subset N_s \)

\( N_x \) Fixed locations of markets

\( N^q_s \) Market locations in region \( q \), \( N^q_s \subset N_s, q \in Q \)

\( N^q_i \) Feasible locations of service centres in region \( q \), \( N^q_i \subset N_s, q \in Q \)

\( L \) Set of facilities, \( L = N_d \cup N_s \)

\( L' \) Set of new facilities, \( L' = N'_s \cup N'_s \)

\( \Gamma \) Arcs in the network, \( \Gamma = \{i, j, t\} \)

\( P_u \) Commercial returns in all periods

\( P_f \) Faulty products in all periods \( P_f \cap P_u = \emptyset \)

\( P_r \) Refurbished products in all periods

\( P'_u \) Used products yielding \( r \in P_r \) as refurbished product

\( S_{pit} \) Supply of commercial return/faulty product \( p \) from market \( i \) at time period \( t \), \( p \in P_u \cup P_f, i \in N_s \)

\( T_{Cijt} \) Transportation cost through arc \( \{i, j, t\} \)

\( V_{wit} \) Capacity to store product/module \( w \) at facility \( i \) during time period \( t \), \( w \in P_r \cup M_u, i \in N_s \cup N_d \)

\( X_{mip} \) Number of modules \( m \) in one unit of faulty product \( p, p \in P_r, m \in M_u \)

\( \beta_{pt} \) Fraction of refurbishable product \( p \) at time period \( t \), \( p \in P_r \)

\( \gamma_{mot} \) Fraction of faulty module \( m \) under warranty, \( m \in M_u \)

\( \lambda_{mut} \) Fraction of faulty module \( m \), \( m \in M_u \)

\( \mu_{mat} \) Fraction of faulty module \( m \) which are sent for repair, \( m \in M_u \)

**Decision Variables**

\( f_{pijt} \) Reverse flow of product \( p \in P_u \cup P_f \) on arc \( \{i, j, t\} \)

\( f'_{pijt} \) Forward flow of product \( p \in P_r \cup P_f \) on arc \( \{i, j, t\} \)

\( e_{mijt} \) Reverse flow of module (under warranty) \( m \in M_u \) on arc \( \{i, j, t\} \)

\( e'_{mijt} \) Forward flow of module (under warranty) \( m \in M_u \) on arc \( \{i, j, t\} \)

\( g_{mijt} \) Reverse flow of module (out-of-warranty) \( m \in M_u \) on arc \( \{i, j, t\} \)

\( g'_{mijt} \) Forward flow of module (out-of-warranty) \( m \in M_u \) on arc \( \{i, j, t\} \)

\( h_{mijt} \) Flow of new module \( m \in M_u \) at time period \( t \)

\( d_{mit} \) Total number of faulty modules \( m \in M_u \) yielded from faulty products at service centre \( i \in N_s \) at time period \( t \)

\( r_{pit} \) Refurbished product \( p \in P_r \) yielded at service centre \( i \in N_s \), at time period \( t \)

\( y_{it} \) Binary indicator of assigning facility to potential location \( i \in L \) at time period \( t \)

\( y'_{it} \) Binary variable to calculate opening cost for new facility \( i \in L' \) at time period \( t \)
Objective Function

Maximize Net Revenue=

Revenue:

\[ \sum_{j \in J} \sum_{N \subseteq \mathcal{N}_j} f_{pji} + \sum_{m \in M} \sum_{i \in I} \sum_{t \in T} \sum_{j \in J} P_i^m J^{pN} R_{mij} \]

+ Transportation cost:

\[ - \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} c_{xy} g_{yz} T_{xy} - \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} c_{yz} g_{xz} T_{yz} + \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} c_{xyz} g_{xyz} T_{xyz} \]

+ Processing cost:

\[ - \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{j \in J} f_{pji} PC_{pji} - \sum_{j \in J} \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} S_{pji} \]

+ Repair cost:

\[ \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} R_{mij} \]

+ Purchasing cost:

\[ \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} h_{mij} B_{pji} \]

+ Disposal cost:

\[ \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} (1 - \beta_j) D_{pji} - \sum_{m \in M} \sum_{i \in I} \sum_{j \in J} D_{mij} (1 - \mu_m) d_{mij} \]

+ Facility cost:

\[ - \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} \sum_{t \in T} F_{xjt} \gamma_y - \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} \sum_{t \in T} \sum_{x = \mathcal{X}} \sum_{y = \mathcal{Y}} \sum_{z = \mathcal{Z}} \sum_{i \in I} \sum_{t \in T} O_{xij} \gamma_y \]

\[ \sum_{j \in J} f_{pji} = \sum_{j \in J} f_{pji} \]

(\forall p \in P, (\forall q \in Q) (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{1})

\[ \sum_{j \in J} f_{pji} = S_{pji} \]

(\forall p \in P, (\forall q \in Q) (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{2})

\[ d_{mij} = \sum_{p \in P} \sum_{i \in I} \sum_{t \in T} \lambda_{mij} X_{pji} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{3})

Constraints:

\[ \sum_{i \in I} \sum_{j \in J} f_{pji} = \sum_{i \in I} \sum_{j \in J} f_{pji} \]

(\forall p \in P, (\forall q \in Q) (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{4})

\[ e_{mij} = \sum_{j \in J} e_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{5})

\[ e_{mij} = \sum_{j \in J} e_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{6})

\[ e_{mij} = \sum_{j \in J} e_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{7})

\[ g_{mij} = \sum_{j \in J} g_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{8})

\[ g_{mij} = \sum_{j \in J} g_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{9})

\[ f_{pji} = f_{pji} \]

(\forall p \in P, (\forall q \in Q) (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{10})

\[ e_{mij} = e_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{11})

\[ g_{mij} = g_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{12})

\[ d_{mij} = \sum_{j \in J} d_{mij} \]

(\forall m \in \mathcal{M}, (\forall j \in \mathcal{N}_j) (\forall t \in T) \tag{13})
Decision variables are network configurations plus forward and reverse flows of products and modules. In other words, the model decide which facility locations to be opened \( \{ \beta_{it}, y'_{it} \} \) and the amount of products and module to be repaired \( \{ e_{mjt}, c_{mij} \} \), refurbished and resold \( \{ f_{pjt}, f'_{pjt} \} \), and disposed \( \{ (1-\beta_{jt})f_{pjt},(1-\mu_m)d_{ne} \} \) at particular regions that maximize the total net profit.

The objective function term (eq.1) maximizes the net profit. The network generates revenue from the sale of refurbished products and repair of the out-of-warranty faulty modules. The costs on the network include transportation, processing, repair, purchasing, disposal, and facility. The transportation cost is incurred due to the product-module movement between the facilities. Processing cost for the commercial returns is the cost of minor refurbishment at service centers. Processing cost for faulty products includes cost to identify faulty modules at service centers. Repair cost is the cost of repairing out-of-warranty faulty modules. Purchasing cost is incurred when the modules are bought from OEMs to meet the repair requirements. Disposal cost is sustained when the network arranges for the disposal of the non-repairable faulty modules or non-refurbishable commercial returns. Facility cost includes the cost of opening new facilities and operating cost of both new and existing facilities.

Constraint sets 2-3 model the flow of commercial returns and faulty products from customers to retailers and service centers. Constraint set 4 models the identification of faulty modules at service centers. Flow of commercial returns from retailers to service centers is modeled in Equation set 5. Constraint sets 6-7 model the flow of faulty modules from service centers to distribution centers. The flow of faulty modules from distribution centers to OEMs and RVs is modeled in Equation sets 8-9. Constraint set 10 models the refurbishment process. Constraint sets 11-12 ensure that the refurbished product demand is satisfied. Equation set 13 balances the flow (forward) of repaired products from service centers to markets with the flow (reverse) of faulty products from markets to service centers. Constraint sets 14-15 balance the flow (forward) of repaired modules from distribution centers to service centers with flow (reverse) of faulty modules between the two. Equation sets 16-17 equate the forward flow of repaired modules from OEMs and RVs to distribution centers with the reverse flow of faulty modules between the two. Constraint set 18 ensures that the repair needs at service centers are satisfied. Equation 19 ensures that the outgoing flow of new modules is equal to the incoming flow of new modules at distribution centers. Constraint sets 20-22 specify the capacity restrictions. Equation set 23 ensures that once a facility is opened, it will be used in the subsequent periods. Constraint set 24 aids in calculation of cost of operation for new facilities. Finally, binary and non-negativity conditions are enforced on variables using constraint sets 25-26.

4. Numerical Example

To validate our mathematical model we test it on a small example problem. Consider a reverse logistics network where two market regions (PM1-PM2) are being served by same number of retailers and service centers. Each retailer and service center serves one of the regions. Four types of commercial returns (P1-P4) are brought by the customers to the retailers (RT1-RT2) who then forward them to the service centers (SC1-SC2) serving them. At service centers, each type of commercial return is converted into its corresponding refurbished product (P9-P12) by carrying out minor refurbishments. The service centers also carry out the repair on four different types of faulty products (P5-P8). Each faulty product is assumed to have three different types of modules with 8 different modules (M1-M8) in total. The faulty modules are identified at the service centers and then forwarded to distribution centers (DC1-DC3) for consolidation. The distribution centers then forward the modules to either OEM or repair vendors (RV1-RV2). It is assumed that the
repair vendors can repair any of the 8 modules. Also, all the modules are manufactured by same OEM.

We implement the model using ILOG OPL Studio 5.1 and solve it using ILOG Cplex 10.1. The objective function value is found to be \(-2,493,800\). The resulting network uses DC1 and DC3 as distribution centers which is due to low supply of commercial returns and the fact that transportation cost from DC2 is considerably higher. RV1 is used for repairing M1, M3, M4, and M5 while RV2 performs the repair for M5, M6, M7, and M8. The network obtained is given in Figure 2.

![Figure 2: Reverse Logistics Network for the Example Problem](https://example.com/figure2)

5. Conclusions and Future Research

A multi-period multi-product reverse logistics network which deals with both commercial returns and faulty products is presented. The optimization model selects optimal facilities and flows that maximize the net revenue. The negativity of objective function implies that income from repairing faulty modules and selling refurbished products cannot cover transportation, facility and costs from after-sales services provided by the company. The MILP model at least gives the highest negative revenues.

Current parameters used in the optimization problems are based largely on the limited available data. The future task is to validate and perform sensitivity analysis with real-life industrial data. The other interesting future work is applying the robust optimization paradigm to the problem. Since our problem is long-term decision making, the decision should be immune to uncertainties. In other words, the decision should perform well inspite of changes in parameters. Parameters such as supply of both commercial and faulty returns, fraction of products in warranty as well as demand for refurbished products are difficult to predict and prone to change in time. The best solution of a current set of parameters might not give the maximum objective in the future. We need to design service network which is not only efficient but robust as well.

### Reference


