A LOCAL SEARCH TECHNIQUE FOR SOLVING A DELIVERY PROBLEM OF FUEL PRODUCTS

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ABSTRACT
This paper addresses on a delivery problem of fuel products, i.e., gasoline, kerosene, and diesel oil, from a supply point to a number of destination points. The delivery is performed using tankers having some compartments. Each compartment is dedicated to a certain fuel product type. A solution of the problem is to determine a tanker’s routing plan for delivering the fuel products in order to fulfill the objective functions with the following priority order: minimizing number of tankers, minimizing total completion time, and minimizing range of tour completion time. The routing problem discussed in this paper is considered as a variant of vehicle routing problems (VRPs) which has the following characteristics: split delivery, multiple routes (multiple trips), and multiple products and compartments. A local search technique is proposed as a solution technique for the problem. The local search technique consists of mechanisms for generating an initial solution and searching candidate solutions. The initial solution is generated using a sequential insertion algorithm. In the searching mechanism, relocation and exchange operators are used to generate candidate solutions. The solution technique is applied in a numerical example of the fuel delivery problem.

KEY WORDS
Vehicle Routing Problems, Split Delivery, Local Search

1. Introduction
As an archipelagic country, delivering fuel products in Indonesia by tankers has an important role. One of the challenges faced by oil companies is how to determine a routing plan for tankers in order to achieve an efficient operation. This routing problem, in general, is known as a vehicle routing problem (VRP) which is introduced initially in [1].

The basic VRP constitutes a single depot acting as a base of vehicles and a number of customers. Each customer has a known delivery demand that must be fulfilled from the depot. The problem deals with determining a vehicle’s routing plan of visiting customers, where each vehicle starts and ends at the depot in order to minimize the total distance. In the basic VRP, it is assumed that there is a single product to be delivered and a vehicle has only one compartment. In addition, each vehicle is assumed only performed one route and a customer is visited only once.

The fuel delivery problem in this paper is considered as a variant of the VRPs which has the following characteristics, i.e., split delivery, multiple routes (multiple trips), and multiple products and compartments. In the split delivery context, one customer can be served by one or more vehicles. In other words, a customer may be visited more than once. The case of multiple routes occurs in the situation when one vehicle may perform more than one route during the planning horizon. It means that a vehicle may leave the depot more than once. Multiple products and compartments characterize a situation when there are several products to be delivered and the vehicles have several compartments where each compartment is dedicated for loading a certain product type.

In literature, the characteristics of the VRP addressed in this paper are discussed separately. The split delivery VRPs have been discussed in [2]-[6]. The discussions of the VRPs with multiple routes appeared in [7]-[15]. The VRPs with multiple products and compartments were presented in [16] and [17].

A construction heuristic based on a sequential insertion algorithm has been proposed in [18] for solving the VRP addressed in this paper. This paper is a further development of [18] by proposing a local search technique.

2. Delivery Problem of the Fuel Products
The real situation of this delivery problem has been described originally in [19]. The problem addresses the
delivering fuel products in East Nusa Tenggara (Indonesia) and East Timor (Figure 1). There are three fuel product types, i.e., gasoline, kerosene, and diesel oil that must be delivered from the supply point in Kupang to the eight destination points, i.e., Atapupu, Dilli, Kalabahi, Larantuka, Maumere, Ende, Reo, and Waingapu.

Tankers are used to deliver the fuel products. In [19], it was assumed that capacities of tanker’s compartments are variable. In this paper, the compartments have fixed capacities. An example of the tanker’s compartment configuration is depicted in Figure 2. In this example, there are ten compartments. Four compartments are dedicated to kerosene as well as diesel oil, and the remainders are used for gasoline.

![Figure 1. Map of East Nusa Tenggara and East Timor](image1)

![Figure 2. Tanker’s compartment configuration](image2)

3. Definition of the Routing Problem

The routing problem discussed in this paper follows the feature of the basic VRP such as single depot and delivery activity. In order to cope the real situation of the fuel delivery problem, the VRP includes the following additional characteristics such as split delivery, multiple routes, and multiple products and compartments. In the following sections, the terminologies related to the VRP under discussion are defined.

**Depot and Customers.** The problem consists of a single depot and a number of customers. The depot acts as the supply point and the base of vehicles. The customers represent the destination points. The depot and customer points are denoted by locations.

**Vehicles.** Vehicles represent tankers. It is assumed that the vehicles are unlimited in number and identical. A vehicle has a number of compartments, where each compartment has a capacity and dedicated to a certain product type.

**Product Type.** Each product type represents the fuel product type to be delivered from the depot to the customers. Demands for each product type in any customer are known within the planning horizon.

**Time between Locations.** The time between two locations represents the time required for vehicles to travel from a location to another location. It is assumed that the time between two locations is symmetrical and constant.

**Loading and Unloading Time.** The loading time is the time required to load products into vehicles at the depot. The unloading time represents the time needed for discharging products from vehicles at the customer points. The loading and unloading times per unit for any type of products are assumed to be equal and constant.

**Route.** A route is defined as a sequence of visiting customers, starting from and ending at the depot. A route contains at least one customer. For each product type, quantities of loads in any route must be lower than or equal to the compartment capacities.

**Tour.** A tour is defined as a sequence of route. A tour has at least one route. The length of tour completion time for any tour must not be greater than the length of planning horizon.

**Planning Horizon.** The planning horizon represents a cycle time of the delivery activity.

**Number of Vehicles.** The number of vehicles represents the total number of vehicles required for performing tours. Each vehicle only performs one tour. Therefore, the number of vehicles is equal to the number of tours.

**Tour Completion Time.** A tour completion time depicts the total time spent by a vehicle for performing a tour. In any tour, it consists of total travel time, total loading time at the depot, and total unloading time at customers. The
Routing Plan. A routing plan represents the plan of vehicles in order to serve tours. The plan consists of information about the sequence of visiting locations by vehicles during the planning horizon. The routing plan also provides information about quantities of each product to be loaded into vehicles at the depot and discharged at the destination points.

Solution of the Routing Problem. A solution of the routing problem is to determine a vehicle’s routing plan for delivering the products in order to fulfill the objective functions with the following priority order: minimizing number of vehicles, minimizing total completion time, and minimizing range of tour completion time without violating vehicle’s compartment capacity and the planning horizon constraints.

4. Mathematical Model

4.1. Notations

Notations used in the model and solution technique are as follows:

- \( i \) : Location index (\( i = 0 \) is the depot, \( i = 1, ..., n \) are customers)
- \( j \) : Product type index
- \( t \) : Tour index (\( t = 1, 2, ... \))
- \( r \) : Route index (\( r = 1, 2, ... \))
- \( k \) : Position index (\( k = 1, 2, ... \))
- \( M \) : Number of product types
- \( NT \) : Number of tours
- \( NR[t] \) : Number of routes in tour \( t \)
- \( NL[t, r] \) : Number positions in route \( r \) of tour \( t \)
- \( L[t, r, k] \) : Location in position \( k \) in route \( r \) of tour \( t \)
- \( w[t, r, j] \) : Quantity of loads in route \( r \) of tour \( t \) for product \( j \)
- \( q[L[t, r, k], j] \) : Quantity of demands for product type \( j \) for location placed in position \( k \) in route \( r \) of tour \( t \)
- \( \tau[L[t, r, k], L[t, r, k + 1]] \) : Travel time from a location placed in position \( k \) in route \( r \) of tour \( t \) to the another location in position \( k + 1 \) in route \( r \) of tour \( t \)
- \( CT[t] \) : Completion time of tour \( t \)
- \( s \) : Loading time per unit of product type
- \( h \) : Unloading time per unit of product type

- \( PH \) : Length of planning horizon
- \( Q[j] \) : Vehicle’s compartment capacity for product type \( j \)
- \( NV \) : Number of vehicles required
- \( TCT \) : Total tour completion time
- \( RCT \) : Range of tour completion time

4.2. Mathematical Formulation

The routing problem addressed in this paper is formulated mathematically as follows.

Minimize

\[
Z = \{NV, TCT, RCT\}
\]

subject to

\[
\begin{align*}
L[t, r, 1] &= 0; \\
&\quad t = 1, ..., NT; r = 1, ..., NR[t] \quad (4.2) \\
L[t, r, NL[t, r]] &= 0 \\
&\quad t = 1, ..., NT; r = 1, ..., NR[t] \quad (4.3) \\
L[t, r, k] &= i; \\
&\quad t = 1, ..., NT; r = 1, ..., NR[t]; k = 2, ..., NL[t, r] - 1; i = 1, ..., n \quad (4.4) \\
w[t, r, j] &\leq Q[j]; \\
&\quad t = 1, ..., NT; r = 1, ..., NR[t]; j = 1, ..., M \quad (4.5) \\
w[t, r, j] &= \sum_{k=2}^{NL[t, r]-1} q[L[t, r, k], j] \\
&\quad t = 1, ..., NT; r = 1, ..., NR[t]; j = 1, ..., M \quad (4.6) \\
CT[t] &\leq PH; t = 1, ..., NT \quad (4.7) \\
CT[t] &= \sum_{r=1}^{NR[t]} \sum_{k=1}^{NL[t,r]} \tau[L[t, r, k], L[t, r, k + 1]] \\
&\quad + (s+h) \sum_{r=1}^{NR[t]} \sum_{j=1}^{M} w[t, r, j] \quad (4.8) \\
NV &= NT \quad (4.9) \\
TCT &= \sum_{t=1}^{NT} CT[t] \quad (4.10)
\end{align*}
\]
5. Solution Technique

5.1. Initial Solution

The initial solution is generated using a sequential insertion algorithm developed in [18]. In the sequential insertion algorithm, a tour for a vehicle is constructed one at a time. For the current tour, the first route is initialized. Customers are inserted to the current route one by one in the position giving a minimum tour completion time. If there is no compartment capacity available, a new route is constructed. When a tour completion time of the vehicle has reached the length of the planning horizon, a new tour is constructed. The algorithm ends if all customer demands already assigned.

By utilizing the nature of the sequential insertion algorithm, the number of vehicles required is possibly small because the algorithm tries to use all capacities. Applying the criteria for the insertion, the minimum tour total completion time is possibly achieved. In the algorithm, the objective of minimizing range of tour completion time has not been considered. The following description gives steps of the sequential insertion algorithm for the VRP discussed in this paper.

Step 0:
Initialize
\[ N = N \quad (N \text{ is the set of unassigned customers}) \]
\[ NT = 0 \]
\[ NV = 0 \]

\[ RCT = \max_i \{CT[i]\} - \min_i \{CT[i]\}; \quad t > 1 \quad (4.11) \]

Equation (4.1) is the overall objective function, i.e., minimizing number of vehicles, minimizing total completion time, and minimizing the range of tour completion time. The equation (4.1) also shows the priority order of fulfilling the objective functions.

Equation (4.2) and (4.3) ensure that the first and last position in any tour and route is the depot. Equations (4.4) ensure that the positions in any tour and route except in the first and last are customers. These equations specify that one customer may appear in one or more positions. This is a nature of the split delivery VRP.

Inequalities (4.5) define vehicle’s compartment capacity constraints. Equations (4.6) determine vehicle’s loads for each product in any route. Inequalities (4.7) represent the tour completion times. Equation (4.8) determines the number of vehicle required. Equation (4.10) specifies the total tour completion time. Equation (4.11) is used for defining the range of tour completion time.

Step 1:
Set
\[ t = 1 \]
\[ r = 1 \]
\[ NT = NT + 1 \]
\[ NR[t] = 1 \]
\[ NL[t, r] = 2 \]
\[ L[t, r, 1] = L[t, r, NL[t, r]] = 0 \]
\[ w[t, r, j] = 0, \quad \forall j \]
\[ CT[t] = 0 \]

Go to step 2.

Step 2:
For \( i \in N \), try to insert \( i \) between positions \( k, k+1 \) for \( k = 1, ..., NL[t, r] - 1 \). If there is no feasible insertion regarding to the planning horizon, go to step 8. Otherwise, select \( i^* \) giving the shortest tour completion time \( CT[t] \) and go to step 3.

Step 3:
If \( q[i^*, j] \leq Q[j] \) for \( \forall j \), then set \( N = N - \{i^*\} \). Set
\[ Q[j] = Q[j] - q[i^*, j], \quad \forall j \]
\[ w[t, r, j] = w[t, r, j] + q[i^*, j], \quad \forall j \]
\[ NL[t, r] = NL[t, r] + 1 \]
\[ L[t, r, 1] = L[t, r, NL[t, r]] = 0 \]
\[ L[t, r, NL[t, r] - 1] = i^* \]

Update \( CT[t] \). Go to step 4.

Step 4:
If \( N = \emptyset \), go to step 5. Otherwise, go to step 9.

Step 5:
For \( i \in N \), try to insert \( i \) between positions \( k, k+1 \) for \( k = 1, ..., NL[t, r] - 1 \). If there are feasible insertions with respect to the planning horizon and compartment capacity constraints, go to step 6. If there are feasible insertions regarding to the planning horizon, but infeasible regarding to compartment capacities, go to step 7. If there is no feasible insertion regarding to the planning horizon, go to step 8.

Step 6:
Select \( i^* \) and the insertion position \( (k^*, k^* + 1) \) giving the shortest tour completion time \( CT[t] \). If \( q[i^*, j] \leq Q[j] \) for \( \forall j \), then set \( N = N - \{i^*\} \). Set
\[ NL[t, r] = NL[t, r] + 1 \]
\[ Q[j] = Q[j] - q[i^*, j], \quad \forall j \]
\[ w[t, r, j] = w[t, r, j] + q[i^*, j], \quad \forall j \]
\[ L[t, r, k^*] = i^* \]

Update the sequence of positions \( L[t, r, m] \) for \( m = k^* + 1, \ldots, NL[t, r] \). Update \( CT[t] \). Go to step 4.
5.2. Local Search Technique

Based on an initial solution, the local search technique is conducted by searching its neighborhoods. In this paper, a searching mechanism is carried out by applying two operators, i.e., relocation (1, 0) and exchange (1, 1) operators. The relocation (1, 0) operator works by relocating a customer from one position to another position, while the exchange (1, 1) operator performs an exchange between two customers from different positions. A solution from the neighborhoods is accepted as the new current best solution if the solution has objective values with the following priority order: smallest number of vehicles, shortest total tour completion time, and shortest range of tour completion time.

The searching mechanism in the local search is conducted by firstly performing the relocation (1, 0) operator and continued by the exchange (1, 1) operator. Under the application of the best improvement strategy in searching candidate solutions, the steps of the local search technique in this paper is described as follows.

Step 0: Create an initial solution \( \theta^0 \) using the sequential insertion algorithm and set the initial solution \( \theta^0 \) as the current best solution \( \theta^0 \), i.e., \( \theta^0 = \theta^0 \). Go to step 1.

Step 1: Set \( \theta = \theta^0 \). Go to step 2.

Step 2: Based on \( \theta^0 \), perform all possible relocations using the relocation (1, 0) operator. If there are feasible solutions, then select the best new solution and denote as \( \theta^1 \) and set it as the current best solution, i.e., \( \theta = \theta^1 \) and set \( \theta^0 = \theta \). Continue this step until there is no feasible solution or no improvement in the objective function. Go to step 3.

Step 3: Based on \( \theta^0 \), perform all possible exchanges using the exchange (1, 1) operator. If there are feasible solutions, then select the best new solution and denote as \( \theta^2 \) and set it as the current best solution, i.e., \( \theta = \theta^2 \) and set \( \theta^0 = \theta^2 \). Continue this step until there is no feasible solution or no improvement in the objective function. Go to step 4.

Step 4: If the objective function of \( \theta^1 \) is better than the objective function of \( \theta^0 \), go to step 1. Otherwise, go to step 5.

Step 5: Stop and set the current best solution \( \theta^* \) as the best solution of the problem \( \theta^* \), i.e., \( \theta^* = \theta^* \).

6. Numerical Example

The solution method is applied in a following numerical example. There is a single supply point (depot) denoted by “A” and eight destination points (customers) denoted by “B” until “I”. Distances between points (locations) are shown in Table 1. The planning horizon is seven days. There are three types of products named as product 1, 2, and 3.

Demands for each fuel product types within the planning horizon are shown in Table 2. A tanker has a total capacity of 1500 kiloliters consisting of three compartments with capacities are 600 (fuel product 1), 600 (fuel product 2), and 300 (fuel product 3) kiloliters. The vehicle’s speed is assumed as 10 knots. Loading and unloading times are set as 200 kiloliters per hour. Setup times at a supply point is assumed as 2 hours.

Table 3 shows the initial solution. In the initial solution, number of tankers required is 3 units, total tour completion time is 478.80 hours, and range of tour completion time is 3.60 hours. The solution using the local search technique is shown in Table 4. It can been seen that number of tankers is equal to the initial solution. Total tour completion is 364.60 hours which is shorter than initial solution. The range of tour completion time is 17.25 hours which is longer than the initial solution. This happens because the objective of minimizing total
completion time has a higher priority than the objective of minimizing range of tour completion time.

7. Conclusion

This paper has discussed a proposed solution method based on the local search technique for solving an example of practical problems of delivering fuel products from a supply point to destination points. The problem is considered as a variant of the VRP, i.e., split delivery, multiple trips, and multiple products compartments. The routing problem is formulated as a multiobjective optimization problem. A further study can be conducted by including additional characteristics that have not been considered in order to give more precise descriptions of the real situation.

<table>
<thead>
<tr>
<th>Table 1: Distance Between Ports (in miles)</th>
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</thead>
<tbody>
<tr>
<td>A 118</td>
</tr>
<tr>
<td>B 180</td>
</tr>
<tr>
<td>C 140</td>
</tr>
<tr>
<td>D 130</td>
</tr>
<tr>
<td>E 218</td>
</tr>
<tr>
<td>F 348</td>
</tr>
<tr>
<td>G 140</td>
</tr>
<tr>
<td>H 201</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Demands for Each Product (in kiloliters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destination Point</td>
</tr>
<tr>
<td>--------------------</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
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<tr>
<td>H</td>
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<table>
<thead>
<tr>
<th>Table 3: Initial Solution</th>
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<tbody>
<tr>
<td>Tanker</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Number of tankers = 3
Total tour completion time = 478.80 hours
Range of tour completion time = 3.60 hours
Table 4: Local Search Solution

<table>
<thead>
<tr>
<th>Tanker</th>
<th>Routing</th>
<th>Tour completion time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A (117.00, 515.80, 191.00) – C (117.00, 515.80, 191.00) – A (276.75, 600.00, 264.00) – E (210.00, 210.00, 105.00) – C (66.75, 390.00, 159.00) – A (0.00, 0.00, 0.00)</td>
<td>114.15</td>
</tr>
<tr>
<td>2</td>
<td>A (469.00, 592.20, 279.30) – I (196.00, 280.00, 119.00) – H (273.00, 312.20, 160.30) – A (241.50, 344.40, 219.10) – F (241.50, 344.40, 219.10) – A (0.00, 0.00, 0.00)</td>
<td>119.06</td>
</tr>
<tr>
<td>3</td>
<td>A (513.10, 442.40, 168.00) – D (112.00, 105.00, 49.00) – G (401.10, 337.40, 119.00) – A (371.00, 378.00, 287.00) – B (371.00, 378.00, 287.00) – A (0.00, 0.00, 0.00)</td>
<td>131.40</td>
</tr>
</tbody>
</table>

Number of tankers = 3 units
Total tour completion time = 364.60 hours
Range of tour completion time = 17.25 hours

References