ABSTRACT
This paper investigates the vehicle routing problem with manual materials handling (VRPMMH). Briefly, when a delivery vehicle arrives at the customer, delivery crew must move selected goods from the vehicle to the customer’s stock room. Knowing the number of customers, required workloads, and number of vehicles, the problem objective is to determine ergonomic work schedules of the delivery crew such that their daily energy expenditure will not exceed the recommended level (i.e., 33% of the energy capacity). Mathematical models for the VRPMMH are developed to find a minimum number of utilized vehicles, their delivery route, and daily work schedules (the number of customers to be served en route) of the crew such that a maximum daily energy expenditure is minimized.

KEY WORDS
Vehicle Routing, Ergonomics, Manual Materials Handling, Workforce Scheduling

1. Introduction
In the classical capacitated vehicle routing problem (VRP), goods are delivered from a depot to a set of customers using a set of identical delivery vehicles. Each customer demands a certain quantity of goods and the delivery vehicles have a limited capacity. Typically, the problem objective is to find delivery routes starting and ending at the depot that minimize a total travel distance without violating the capacity constraint of the deliver vehicles. In some VRPs, the problem objective might be to determine the minimum number of delivery vehicles to serve all customers.

The classical VRP is defined on an undirected graph \( G = (N, E) \), where \( N = \{0, 1, 2, \ldots, n\} \) is a node set and \( E = \{(i, j) : i, j \in N\} \) is an edge set. For simplicity, \((i, j)\) and \((j, i)\) represent the same edge. Node 0 corresponds to a depot at which \( m \) identical vehicles of capacity \( C \) are based, while the remaining nodes are customers. Each customer \( i \) has a nonnegative demand \( q_i \). With each edge \((i, j)\) is associated a cost \( c_{ij} \) corresponding to a distance or to a travel time. The VRP consists of determining vehicle routes of a minimum total cost satisfying the following constraints:

1. Each route starts and ends at the depot.
2. Each customer belongs to exactly one route.
3. The total customer demand of any route does not exceed \( C \).
4. The total cost of any route does not exceed a preset limit \( D \).

The VRP is NP-hard and can rarely be solved exactly for values of \( n \) in excess of 100 [1].

There are many variants of VRP such as the vehicle routing problem with backhauls (VRPB), the pickup and delivery problem with time windows (VRPTW), the mixed vehicle routing problem with backhauls (MVRPB), the multiple depot mixed vehicle routing problem with backhauls (MDMVRPB), the vehicle routing problem with backhauls and time windows (VRPBTW), the mixed vehicle routing problem with backhauls and time windows (MVRPBTW), and the vehicle routing problem with simultaneous deliveries and pickups (VRPSDP). For the detailed explanation of each problem, see Ropke and Pisinger [2].

Specifically, the vehicle routing problem dealt with in this paper falls in the same category as the vehicle routing problem with simultaneous deliveries and pickups (VRPSDP). All or some of the customers simultaneously demand goods from (and supply goods to) the depot, and thus both a delivery and a pickup should occur at these customers. The pickup and delivery should be performed simultaneously such that each customer is visited only once by a vehicle. The typical problem objective is to minimize the total distance. The VRPSDP was first introduced by Min [3] to solve the problem of transporting books between public libraries and a library administration center (acting as a depot). Halse [4] presented exact and heuristic methods for the problem and Dethloff [5, 6] considered heuristic algorithms. Salhi and Nagy [7] used their MVRPB heuristic to solve the
problem. However, the simultaneous “deliveries and pickups” constraint is not handled by their heuristic.

When the delivery vehicle arrives at the customer, delivery crew must move selected goods from the vehicle to the customer’s stock room. Such manual materials handling requires the crew to expend their physical energy. The amount of energy required at each customer location depends on the workload to be performed. If the total workload is too strenuous, the crew are likely to develop excessive muscular and whole-body fatigue, which can cause occupational accidents and over-exhaustion after work. Ergonomic workforce scheduling is recommended as a means to reduce the risk of over-exhaustion [8]. Generally when assigning a worker to perform a physical task, it is necessary to know the amount of energy required to perform the task (or energy cost) and the energy capacity of the worker (or energy limit/budget). Based on an ergonomic recommendation, the daily (8-hr workday) energy expenditure of the worker should not exceed 33% of his/her energy capacity [8]. (For simplicity, we shall refer to this recommended capacity as the working energy capacity.) For different individuals, their working energy capacities normally vary. The number of physical tasks to be safely assigned to any worker thus depends on both the energy costs of individual tasks and the worker’s working energy capacity.

Yaoyuenyong and Nanthavanij [9] proposed the energy-based workforce scheduling problem (WSP-E) to determine the worker-task-period assignments for $m$ workers and $n$ tasks such that the number of workers is minimized and, for each worker, the total energy cost does not exceed the daily working energy capacity. Since the length of work period is constant for practical management, WSP-E is a discrete scheduling problem. Efficient algorithms for determining an optimal set of worker-task-period assignments for WSP-E were also discussed [9].

In this paper, we approach the vehicle routing problem with manual materials handling (VRPMMH) from an ergonomic viewpoint and attempt to find a set of customers to be safely assigned to each delivery vehicle (and its crew) such that the total workload does not exceed the sum of the working energy capacity of the crew. That is, we view the VRPMMH as a variant of the WSP-E. Two mathematical models are proposed to represent the VRPMMH. The models are solved by LINGO (an optimization program) to find the minimum number of vehicles and their delivery routes such that the maximum total energy expenditure is minimized.

2. Mathematical Models

Let us consider the vehicle routing problem with $m$ delivery vehicles and $n$ customers. Initially, goods are loaded on each delivery vehicle at the depot to be delivered to customers along the assigned route. When arriving at the customer location, the delivery crew will unload the goods from the vehicle, move them into a stock room, pickup returned goods from the stock room, and load them on the delivery vehicle. Materials handling activities at the customer location are performed manually.

2.1 Assumptions

- Each customer will be served by only one delivery vehicle.
- The number of delivery vehicles is known and fixed.
- The number of delivery crew per vehicle is known and fixed.
- The delivery crew are identical in terms of the working energy capacity.
- The total daily energy expenditure of each delivery crew must not exceed the working energy capacity.
- Travel times from the depot to each customer and between customers are equal.
- Each delivery vehicle must serve all customers en route irrespective of the duration of the work shift.

2.2 Notation

- $c$ number of delivery crew per vehicle
- $e_i$ total daily energy expenditure (kcal/day) of delivery crew of vehicle $i$
- $E$ working energy capacity (kcal/day) of a delivery crew
- $M$ number of available delivery vehicles (fleet size)
- $n$ number of customers
- $r_j$ energy cost (kcal) at customer $j$
- $x_{ij}$ binary variable where $x_{ij} = 1$ if vehicle $i$ is assigned to customer $j$; $x_{ij} = 0$ otherwise
- $y_i$ binary variable where $y_i = 1$ if vehicle $i$ is utilized; $y_i = 0$ otherwise

2.3 Minisum Model

The minisum model is intended to find the minimum number of delivery vehicles to serve all $n$ customers without imposing excessive physical workload on the delivery crew. That is, the total daily energy expenditure of each crew does not exceed $E$. 

Minimize $\sum_{i \in M} y_i$

subject to

$\sum_{j=1}^{n} r_j x_{ij} \leq c \cdot E \quad \forall i \in M$

$\sum_{i \in M} x_{ij} y_i = 1 \quad \forall j$

$x_{ij}, y_i \in (0, 1) \quad \forall ij$

2.4 Minimax Model

The minimax model is intended to find a set of customers for $m$ delivery vehicles (where $m$ is the number of utilized vehicles determined by the minisum model) such that the maximum total daily energy expenditure is minimized. Note that the total daily energy expenditure is the sum of energy costs of the customers en route.

Minimize $Z$

subject to

$\sum_{j=1}^{n} r_j x_{ij} = e_i \quad \forall i$

$e_i \leq Z \quad \forall i$

$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall j$

$x_{ij} \in (0, 1) \quad \forall ij$

3. Numerical Example

Let us consider a single depot, intra-city beverage delivery problem with a fleet of six delivery vehicles and 20 customers. Beverage bottles are stored in plastic cases, with 24 bottles in each case. Initially, beverage cases are loaded on each delivery vehicle at the depot to be delivered to the customers en route. When arriving at the customer location, the delivery crew will unload the beverage cases, move them into the customer’s stock room, pickup cases of empty bottles from the stock room, and load them on the delivery vehicle. Materials handling activities at the customer location are performed manually using hand dolly.

Assuming that there are two crew members per vehicle and each has the working energy capacity of 600 kcal/day, the energy supply per vehicle thus is 1,200 kcal/day. The energy costs are shown in Table 1.

Table 1: Energy Costs ($r_i$’s) of the 20 Customers (kcal/day)

<table>
<thead>
<tr>
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<tr>
<td>C1</td>
<td>330</td>
<td>C6</td>
<td>165</td>
<td>C11</td>
<td>175</td>
<td>C16</td>
<td>210</td>
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<tr>
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<td>475</td>
<td>C7</td>
<td>480</td>
<td>C12</td>
<td>415</td>
<td>C17</td>
<td>405</td>
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<td>215</td>
<td>C8</td>
<td>355</td>
<td>C13</td>
<td>450</td>
<td>C18</td>
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</tr>
<tr>
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<td>320</td>
<td>C9</td>
<td>205</td>
<td>C14</td>
<td>150</td>
<td>C19</td>
<td>335</td>
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<td>C5</td>
<td>180</td>
<td>C10</td>
<td>200</td>
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<td>190</td>
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<td>280</td>
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</table>

The theoretical minimum number of delivery vehicles $V$ for this problem can be approximated from

$V = \frac{\sum_{j=1}^{n} r_j}{c \cdot E}$

That is, at the minimum, the number of delivery vehicles to serve the 20 customers is 5 units.

4. Results

From the minisum model, it is found the minimum number of utilized delivery vehicles for serving the 20 customers is 5 units. Table 2 shows a list of customers (or work schedule), total energy cost (or daily energy expenditure), and residual energy of each vehicle.

It is noted that there is a large variation among the residual energies in Table 2. This result indicates that some vehicle (and its crew) is assigned to the route in which the total workload is heavier than others. More specifically, vehicles V1, V2, and V3 receive heavier workload than the other two vehicles. In order to level the workloads among the vehicles, the minimax model is applied. The revised work schedules are shown in Table 3. (The program is terminated at 6.6 million iterations.) Readers can see that the revised assignment of customers to vehicles is improved since the workload distribution among the 5 vehicles is now more even than earlier.
5. Conclusion

In this paper, we propose the vehicle routing problem with manual materials handling (VRPMMH) in which manual loading/unloading and moving of goods are performed by the delivery crew at the customers. We also develop two mathematical models, \textit{minisum} and \textit{minimax}, for the VRPMMH. These two models are based on the workforce scheduling models for reducing ergonomics hazards.

The \textit{minisum} model is applied to find the minimum number of delivery vehicles to serve all customers so that their workloads do not exceed the recommended energy expenditure constraint. The model also determines the delivery routes for the utilized vehicles. The \textit{minimax} model helps to improve the delivery routes by reassigning the customers to the delivery vehicles in order to level the workloads imposing on all delivery crew.

For future research studies, travel times and working times (at individual customers) will be taken into account. Moreover, the VRPMMH in which delivery crew are non-identical will also be studied.

References

### Table 2: Work Schedules Based on the Minisum Model

<table>
<thead>
<tr>
<th>Route</th>
<th>Vehicle</th>
<th>Customers</th>
<th>Total Energy Cost (kcal/day)</th>
<th>Residual Energy (kcal/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V1</td>
<td>C3, C5, C15, C19, C20</td>
<td>1,200</td>
<td>0</td>
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<tr>
<td>2</td>
<td>V2</td>
<td>C6, C7, C9, C10, C14</td>
<td>1,200</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>V3</td>
<td>C8, C13, C18</td>
<td>1,130</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>V4</td>
<td>C2, C4, C17</td>
<td>1,200</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>V5</td>
<td>C1, C11, C12, C16</td>
<td>1,130</td>
<td>70</td>
</tr>
</tbody>
</table>

### Table 3: Revised Work Schedules Based on the Minimax Model

<table>
<thead>
<tr>
<th>Route</th>
<th>Vehicle</th>
<th>Customers</th>
<th>Total Energy Cost (kcal/day)</th>
<th>Residual Energy (kcal/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V1</td>
<td>C2, C3, C14, C19</td>
<td>1,175</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>V2</td>
<td>C1, C10, C13, C15</td>
<td>1,170</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>V3</td>
<td>C7, C12, C20</td>
<td>1,175</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>V4</td>
<td>C8, C9, C16, C17</td>
<td>1,175</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>V5</td>
<td>C4, C5, C6, C11, C18</td>
<td>1,165</td>
<td>35</td>
</tr>
</tbody>
</table>