Numerical Solutions of the Damped Vibration of a Suspended String

บทคัดย่อ

บทความวิจัยนี้นำเสนอวิธีการใช้ระเบียบวิธีผลต่างอันระเพาะที่ทำผลเฉลยประมาณค่าชิงตัวเลขของ
ปัญหาคำนวณดังกล่าวของสมการการสั่นในแนวตั้งแบบมีความหน่วงชิงส้นและไม่ชิงส้น
ซึ่งทำผลโดยใช้ระเบียบเทียบกับการสั่นแบบไม่มีความหน่วงภายในได้รับความรับกับ
ผลที่ได้แสดงให้เห็นว่าการสั่นแบบมีความหน่วงไม่ชิงส้นผลต่างอย่างมาก
ในขณะที่การสั่นแบบไม่มีความหน่วงชิงส้นผลต่าง
และผลลัพธ์ของการสั่นที่แตกต่างจากจริงปานกลางของเส้นลาดตระกูลมาก
นอกจากนี้ยังพบว่าคำสั่นประสิทธิ์ของความหน่วง
มีผลต่อการหาผลลัพธ์ของการสั่น

คำสำคัญ: สมการการสั่นของเส้นลาดตระกูล แนวตั้ง การสั่นแบบมีความหน่วง วิธีผลต่างอันละ
แบบจำลองชิงตัวเลข

Dr. Jaipong Kasemsuwan
Lecturer, Department of Mathematics
Faculty of Science
King Mongkut’s Institute of Technology Ladkrabang
E-mail: kwjaipon@kmitl.ac.th
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Abstract

This paper presents the use of different finite schemes to find the approximated numerical solutions of the initial-boundary-value problem involving the linear and nonlinear damped equations of a suspended string. The results are compared with the suspended string without a damping term provided that the initial shape and velocities are all the same. The results show that the vibration amplitude decreases and the vibration pattern demonstrated is quite different from the initial pattern in the nonlinear damping case. The vibration amplitude also decreases in the linear damping case but the initial vibration pattern is still maintained. The results are compared graphically and also summarized in Tables. In addition, we find that the coefficient of the damping term plays a role in determining the vibration amplitude.

Keywords: Suspended String Equation, Damped Vibration, Finite-Difference Method, Numerical Simulation

Introduction

In this work, we study the numerical solution of the initial boundary value problem of the suspended string equation with a damping term as follows:

\[
\begin{align*}
  u_{tt} - (xu_{x} + u_{x}) + \alpha |u_t|^{c-1}u_t &= 0 & \alpha > 0, c \geq 1 \\
  u(a,t) &= 0, & t \in [0,T], \\
  u(x,0) &= \phi(x), & u_t(x,0) = \psi(x), & x \in [0,a],
\end{align*}
\]  

(1)

where \( u(x,t) \) is the horizontal displacement of the string at \( (x,t) \). \( \alpha \) is the coefficient of the damping term and a positive number, \( c \geq 1 \). The string being investigated is assumed to be heavy and flexible with the length of \( \alpha \). In addition, the string is assumed to have a uniform density and be suspended with the upper end fixed and the lower end free.

This equation (1) was first introduced by Koshlyakov, Gliner and Smirnov (1964) to explain the vibration of a suspended string.

Eq. (1) was used to describe the energy decay of the global solution and was explained in Kasemsuwan (2009). The time-periodic solution of a damped suspended string equation, taking into account only the linear damping term, was comprehensively studied.
by Yamaguchi, Nagai, and Matsukane (2008). The numerical solution of Eq. (1) without the damping term was obtained using the Crank-Nicolson method as Subklay and Kasemsuwan (2011), and was compared with the finite difference method suggested by Kasemsuwan, Chitsakul and Chaisanit (2010). The characteristics of the several types of solutions previously reported are discussed by Kasemsuwan (2009). The finite difference method is one of the most well known numerical methods for finding the numerical solution of the partial differential equations in the hyperbolic type. For example, Zhang (2005); Kutluay, and Esen (2009); Koide, and Furihata (2009) used this method to find the numerical solution of the regularized long-wave equation. In this work, we apply the finite difference method to find the numerical solution of Eq. (1) considering both linear damping ($c = 1$) and nonlinear damping ($c = 2$) terms. To the best of the author’s knowledge, no attempts have been made to solve Eq. (1) in either case. The numerical solution of a suspended string with both the linear and nonlinear damping cases is compared with the solution without damping under the same initial conditions. We have found that the vibration amplitude is inversely proportional to the $\alpha$ term while the vibration amplitude is proportional to $\alpha$ in the nonlinear damping case. The finite difference method is implemented by using MATLAB and Excel. The results are graphically shown to ease the interpretation and also summarized in Tables.

The Finite Difference Scheme

To solve Eq. (1), we have modified the initial condition so the suspended string equation can be shown as:

$$
\begin{align*}
& u_{tt} - \left((m\Delta x)u_{xx} + u_x\right) + \alpha |u_{t}^{c-1}| u_t = 0, & \alpha < 0, c \geq 1, \\
& u(0,t) = 0, \quad u_x(a,t) = 0, & t \in [0,T], \\
& u(x,0) = \phi(x), \quad u_t(x,0) = \psi(x), & x \in [0,a].
\end{align*}
$$

(2)

The solution domain $0 < x < 1, t > 0$ is divided into subintervals $\Delta x$ in the direction of the position $x$ and time $t$.

Applying the central finite difference to Eq. (2) yields:

$$
\left(\frac{u_{m+1}^{n+1} - 2u_{m}^{n} + u_{m-1}^{n}}{k^2}\right) - \left[mh \left(\frac{u_{m+1}^{n} - 2u_{m}^{n} + u_{m-1}^{n}}{h^2}\right) + \left(\frac{u_{m+1}^{n} - u_{m-1}^{n}}{2h}\right)\right] + \alpha \left|\frac{u_{m+1}^{n+1} - u_{m}^{n+1}}{2k}\right| \left(\frac{u_{m+1}^{n+1} - u_{m}^{n+1}}{2k}\right) = 0,
$$

(3)
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where \( u_m^{n+1} = u(x, t+k) \), \( u_m^n = u(x, t) \), \( u_m^{n-1} = u(x, t-k) \), \( u_m^{n+1} = u(x+h, t) \), \( u_m^{n-1} = u(x-h, t) \), and \( n \) is the time step \( (n = 1, \ldots, N) \) while \( h \) and \( k \) are the mesh size in \( x \) and \( t \), respectively.

In this work, the numerical solution of Eq. (3) is categorized into two different cases, i.e., linear damping case \((c = 1)\) and nonlinear damping case \((c = 2)\).

In the linear damping case, substituting \( c = 1 \) into Eq. (3), we get

\[
\left( \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{k^2} \right) - \left[ mh \left( \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{h^2} \right) + \frac{u_m^{n+1} - u_m^{n-1}}{2h} \right] + \alpha \left( \frac{u_m^{n+1} - u_m^{n-1}}{2k} \right) = 0. \tag{4}
\]

Similarly, substituting \( c = 2 \) into Eq. (3) for the nonlinear damping case yields

\[
\left( \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{k^2} \right) - \left[ mh \left( \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{h^2} \right) + \frac{u_m^{n+1} - u_m^{n-1}}{2h} \right] + \alpha \frac{u_m^{n+1} - u_m^{n-1}}{2k} = 0. \tag{5}
\]

It is noted here that when \( u_m^{n+1} - u_m^{n-1} < 0 \), the numerical solution, i.e. \( u_m^n \), has the same magnitude as when \( u_m^{n+1} - u_m^{n-1} > 0 \) but with the opposite sign.

When multiplying Eq. (4) with \( 2k^2 \), the result is

\[
(2u_m^{n+1} - 4u_m^n + 2u_m^{n-1}) - \left[ 2mp \left( u_m^{n+1} - 2u_m^n + u_m^{n-1} \right) + p \left( u_m^{n+1} - u_m^{n-1} \right) \right] + \alpha k \left( u_m^{n+1} - u_m^{n-1} \right) = 0 \tag{6}
\]

where \( p = \frac{k^2}{h} \), \( h \), \( h \) and \( k \) are the mesh size in \( x \) and \( t \), respectively.

By rearranging Eq. (6) into an explicit form, we have

\[
u_m^{n+1} = \frac{1}{2 + \alpha k} \left[ (2mp + p)u_{m+1}^n + (4 - 4mp)u_m^n + (2mp - p)u_{m-1}^n + (\alpha k - 2)u_m^{n-1} \right]. \tag{7}
\]

Eq. (7) can be subdivided into four different cases, depending on the values of \( n \) and \( m \) as follows:

**Case 1: \( n = 0 \) and \( m = 1, 2, 3, \ldots, M-1 \)**

By substituting \( n = 0 \) into Eq. (7), \( u_m^{-1} \), the outside cylindrical domain, can be estimated by applying the backward difference into the initial condition to obtain:

\[
u_m^{-1} = u_m^0 - k\psi(x). \tag{8}
\]
Substituting Eq. (8) into Eq. (7) yields

\[ u_m^1 = \frac{1}{2 + \alpha k} \left[ (2mp + p)u_{m+1}^0 + (2 - 4mp + \alpha k)u_m^0 + (2mp - p)u_{m-1}^0 + (-\alpha k^2 + 2k)\psi(x) \right]. \] (9)

**Case 2:** \( n > 0 \) and \( m = 1, 2, 3, \ldots, M-1 \)

Eq. (7) is employed.

**Case 3:** \( n = 0 \) and \( m = M \)

In this case, we find that \( u_{M+1}^0 \) is the outside cylindrical domain and can be estimated by applying the central difference into the boundary condition to obtain \( u_{M+1}^0 = u_{M-1}^0 \)

Consequently, Eq. (7) can be simplified to

\[ u_m^1 = \frac{1}{2 + \alpha k} \left[ (2 - 4Mp + \alpha k)u_m^0 + 4Mp u_{M+1}^0 + (-\alpha k^2 + 2k)\psi(x) \right]. \] (10)

**Case 4:** \( n > 0 \) and \( m = M \)

In this case, \( u_{M+1}^n \) can be estimated by applying the central difference into the boundary condition in a similar manner to Case 3 and we obtain

\[ u_m^{n+1} = \frac{1}{2 + \alpha k} \left[ (4 - 4Mp)u_m^n + 4Mp u_{M+1}^n + (\alpha k - 2)u_M^n \right] \] (11)

The numerical solutions of Eq. (1) for the linear damping case \( (c = 1) \) can be calculated by applying the finite difference equations to Eqs. (7), (9), (10) and (11).

For the nonlinear damping case \( (c = 2) \), we apply \( \left| u_m^{n+1} - u_m^{n-1} \right|^2 \) to Eq.(5) and we have

\[ \frac{\alpha}{2(2 - \alpha u_m^{n-1})}(u_m^{n+1})^2 + u_m^{n+1} = \frac{1}{(2 - \alpha u_m^{n-1})}\left[ (2mp + p)u_{m+1}^n + (4 - 4mp)u_m^n + (2mp - p)u_{m-1}^n - (\alpha u_m^{n-1} + 2)u_m^{n-1} \right]. \] (12)

Similarly, Eq. (12) can be treated in the same way as Eq. (7), in other words. Eq. (12) can be categorized into 4 different cases depending on the values of \( n \) and \( m \). Subsequently we obtain the finite difference schemes to find the numerical solutions of Eq. (1) for \( c = 2 \) as follows:
Numerical Solutions of the Damped Vibration of a Suspended String

**Case 1:** $n = 0$ and $m = 1, 2, 3, ..., M-1$

\[
\frac{\alpha}{2\left(2 - \alpha (u_m^0 - kf_2(x))\right)} \left((u'_m)^2 + u_m'\right) = \frac{1}{\left(2 - \alpha (u_m^0 - kf_2(x))\right)} \left[(2mp + p)u_{m+1}^0 + (4 - 4mp)u_m^0 + (2mp - p)u_{m-1}^0 \right]
\]

\[
\left[-(\alpha (u_m^0 - kf_2(x)) + 2)(u_m^0 - k\psi(x))\right].
\]

(13)

**Case 2:** $n > 0$ and $m = 1, 2, 3, ..., M-1$

\[
\frac{\alpha}{2\left(2 - \alpha u_m^{n-1}\right)} \left((u_m^n)^2 + u_m^n\right) = \frac{1}{\left(2 - \alpha u_m^{n-1}\right)} \left[(2mp + p)u_{m+1}^n + (4 - 4mp)u_m^n + (2mp - p)u_{m-1}^n - (\alpha u_m^{n-1} + 2)u_m^n \right]
\]

(14)

**Case 3:** $n = 0$ and $m = M$

\[
\frac{\alpha}{2\left(2 - \alpha (u_M^0 - kf_2(x))\right)} \left((u_M^1)^2 + u_M^1\right) = \frac{1}{\left(2 - \alpha (u_M^0 - kf_2(x))\right)} \left[4Mpu_{M-1}^0 + (4 - 4Mp)u_M^0 \right]
\]

\[
\left[-(\alpha (u_M^0 - kf_2(x)) + 2)(u_M^0 - k\psi(x))\right].
\]

(15)

**Case 4:** $n > 0$ and $m = M$

\[
\frac{\alpha}{2\left(2 - \alpha u_M^{n-1}\right)} \left((u_M^n)^2 + u_M^n\right) = \frac{1}{\left(2 - \alpha u_M^{n-1}\right)} \left[4Mpu_{M-1}^n + (4 - 4Mp)u_M^n - (\alpha u_M^{n-1} + 2)u_M^n \right].
\]

(16)
Results and Discussion

The numerical solutions of the suspended string vibration with linear and nonlinear damping are shown using the finite difference in Tables 1 and 2, respectively. The results are obtained under the same initial shape (i.e. $\sin (2\pi t)$) without the initial velocity.

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The numerical simulations considering both non-damping (blue line) and linear damping cases (red line) under the same initial shape (i.e. $\sin(2\pi \nu)$) and the various $\alpha$ i.e. $\alpha = 1, 10,$ and $30$ are illustrated in Figures 1-3 for different values of $t$. They reveal that when alpha increases, the vibration amplitude decreases. In our calculations, we have varied the value of alpha from 0.1 to 10,000 and found that the vibration amplitude gradually decreases and approaches the initial shape of the suspended string as shown in Figure 4.

**Figure 1** Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Linear Damping (Red Line) for Different Values of Alpha and Time (without the Initial Velocity)

**Figure 2** Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Linear Damping (Red Line) for Different Values of Alpha and Time (the Initial Velocity: 1 m/s)
Figure 3  Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Linear Damping (Red Line) for Different Values of Alpha and Time (the Initial Velocity: \( x \) m/s)

Figure 4  Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Linear Damping (Red Line) for Different Values of Alpha and Time (the Initial Velocity: \( x \) m/s)

The numerical simulation of a suspended string compared between non-damping and nonlinear damping cases under the same initial shape (i.e. \( \sin(2\pi t) \)) and various \( \alpha \) i.e. \( \alpha = 0.5, 2.5, 3.1 \) are illustrated in the following Figures 5 and 6 to show that as the value of alpha decreases, the vibration amplitude decreases. In our calculations, we have varied alpha from 3.1 to 3.5 and the amplitude is gradually increased until the resonance is developed as shown in Figure 7. In addition, the initial velocity was varied and the numerical solution shows a little variation in the vibration amplitude with nonlinear damping provided that the values of alpha and the initial shape are the same. This fact is confirmed in Figures 3 and 4.
5 and 6. From Figures 5 and 6, when the initial velocity is varied from zero to one and the nonlinear damping term is taken into account, the vibration amplitude remains almost the same (see red line). However, the vibration amplitude demonstrates some variation occurs when the damping term is totally ignored (see blue line).

**Figure 5**  Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Nonlinear Damping (Red Line) for Different Values of Alpha and Time (without the Initial Velocity)

**Figure 6**  Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Nonlinear Damping (Red Line) for Different Values of Alpha and Time (the Initial Velocity: 1 m/s)
Comparing results shown in Figures 5 and 6 show that the nonlinear damping term plays a more important role in decreasing the vibration amplitude than the linear damping term. In the linear damping case, the vibration amplitude is inversely proportional to the term $\alpha$, while the vibration amplitude is proportional to $\alpha$ in the nonlinear damping case. Furthermore, we investigated the vibration characteristics under various initial constant velocities. We found that the vibration characteristics maintain the nondamped shape in the linear damping case, while the vibration characteristics do not depend on the nondamped shape in the nonlinear damping case, as illustrated in Figure 8. From Figure 8, the resonance takes place when the damping term is ignored (blue line) and this resonance is still preserved if the linear damping term is included mainly due to the fact that the vibration maintains the initial resonance shape. However, it is noticed that the resonance is attenuated when the nonlinear damping term is included.
Figure 8  Graphical Comparison of the Vibration Displacements without Damping (Blue Line) and with Linear and Nonlinear Damping (Red Line) under the Same Initial Velocity 3 and $\alpha = 0.5$

The stability condition of the finite difference scheme is $2mp < 1$ where $p = \frac{k^2}{h}$ with $h = 0.02$ and $k = 0.0025$. We cannot limit study to more nonlinear damping cases i.e. $c > 2$ with various velocities, but also external forces. Other numerical methods also need to be studied to compare the obtained results with the finite difference method.

Conclusion

The numerical solution of the suspended string equation with linear and nonlinear damping is obtained using the finite difference method. The compared results, taking into account both the linear and nonlinear cases, are shown to be quite different provided that the same initial shape and initial velocities are assumed. The results obtained in the linear damping case shows the vibration, which maintains the non-damped shape, while the vibration characteristic shows quite a different shape in the case of nonlinear damping. In addition, in the linear damping case, the vibration amplitude is inversely proportional to the coefficient of the damping term, while the vibration amplitude is proportional to the coefficient of the damping term in the nonlinear damping case. Furthermore, the nonlinear damping term prevents resonance from occurring under the defined coefficient of the damping term.

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Dr. Jaipong Kasemsuwan is a lecturer in Department of Applied Mathematics, Faculty of Science, King Mongkut’s Institute of Technology Ladkrabang (KMITL), Bangkok, Thailand. Her research interest includes partial differential equation, especially suspended string equation, functional analysis. She received her Ph.D. of Mathematics and Mathematical Science from Tokai University, Japan.